

UNIVERSAL ACTIVE MATH  
Multiplication  
A Systematic Structured Method  
Based On the Principle that in Things Language

Every Multiplication gives a rectangle. Every rectangle is a multiplication.

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Important points about Universal Active Math


1. Learning by understanding and learning only by understanding.
  2. A systematic pedagogy based on the Chinese proverb :  
“ I hear, I forget; I see, I remember; I do, I understand”
  3. Things language understanding first; then the alphanumeric language.
  4. Systematic translation from things language to alphanumeric language.
  5. Support, monitoring, assessment systems to ensure that every teacher and every child becomes comfortable with mathematics.
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MULTIPLICATION IN THINGS LANGUAGE :

Use Jodo blocks / marbles / pebbles for the following sequence :

\* Give me  $2 + 3$ .

With Jodo Blocks : Children will make : 

With Pebbles : Children will give : 

2 pebbles of one colour and 3 of other colour.  
OR 2 in one hand and 3 in the other hand and then  
joined together.

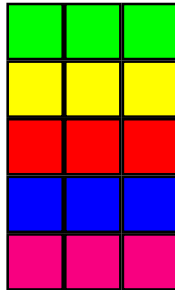
\* Give me  $3 + 3$ .



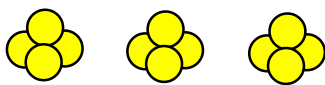
\* Give me  $3 + 3 + 3 + 3 + 3$



- \* When the same number is added (joined) to itself repeatedly we can also join the groups of 3 blocks differently, one below the other to make a rectangle.  $3+3+3+3+3$

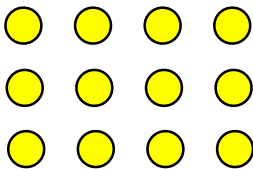


- \* Bhajiwala has made piles of 4 lemons each.



This also has a multiplication :  
Three times four :  $3 \times 4$

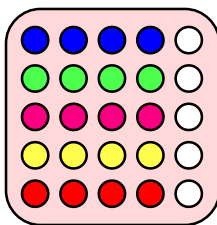
The lemons can also be arranged thus to make a rectangle :



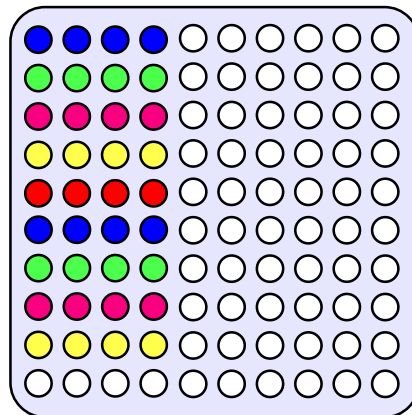
THUS WE CAN SAY THAT  
EVERY MULTIPLICATION GIVES A RECTANGLE  
AND  
EVERY RECTANGLE IS A MULTIPLICATION.

Use mathemat and Jodo Blocks to make rectangles to represent multiplication.

$$5 \times 4$$



$$9 \times 4$$



All multiplication tables upto  
 $10 \times 10$  can be made on the mathemat.

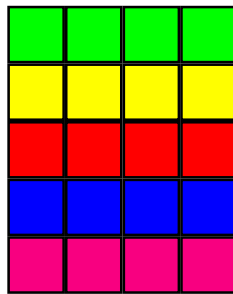
When children learn multiplication as making rectangles they learn to connect :

- multiplication with the associated rectangle.
- the number with its factors.
- the rectangular shape with the number.
- repeated addition and multiplication to the rectangle.
- multiplication with area

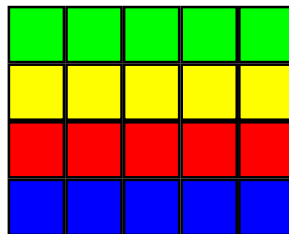
## THE LINGUISTIC PROBLEM IN MULTIPLICATION

Pay attention to the linguistic problem!

$5 \times 4$  is read in English as 'five times four' and indicates five groups of four each.



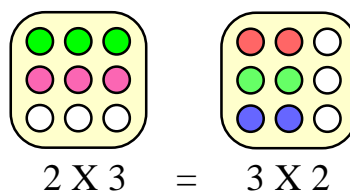
But  $5 \times 4$  is read in Marathi or Hindi as 'paanch guna char' which means four groups of five each.



This write up uses the English language interpretation.

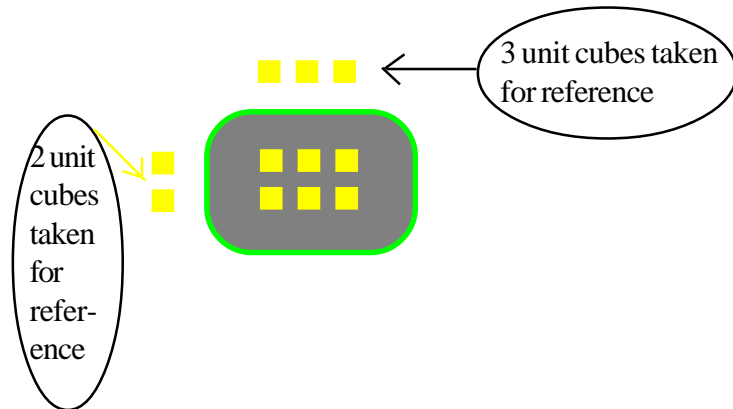
- \* Ask children to make  $2 \times 3$  ( Two times three. Two groups of three each).
- \* Ask them to make  $3 \times 2$  (Three times two. Three groups of two each).

Children can verify for themselves that these are equal :



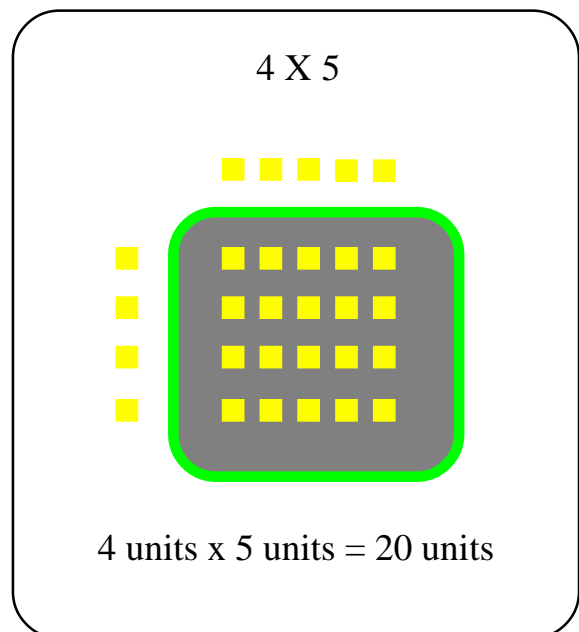
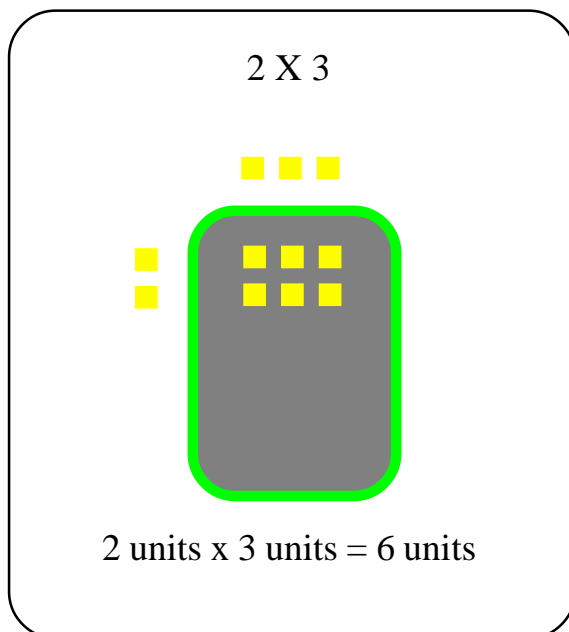
USE DIENES' BLOCKS (UTH kit) FOR FURTHER MULTIPLICATIONS.

$2 \times 3$  : Two times three. Three taken two times



Children make a rectangle of  $2 \times 3$  on their slate placing unit cubes as above.

UNITS TIMES UNITS GIVES UNITS



$$U \times U = U$$

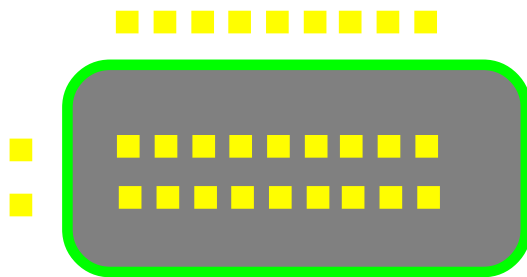
Children make rectangles with dienes' blocks and discover the rule  $U \times U = U$

(Of course, subsequently the 20 unit cubes can be joined to make 2 tens)

Note : While solving these problems children are not writing multiplication in alphanumeric language. They are making shapes and recognizing (counting) the result.

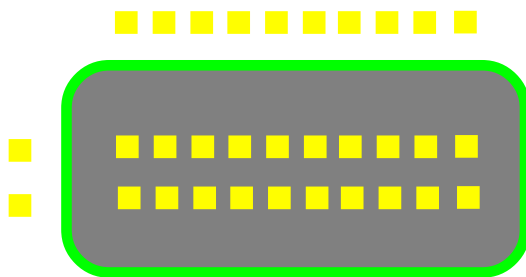
## UNITS TIMES TENS IS TENS

$2 \times 9$



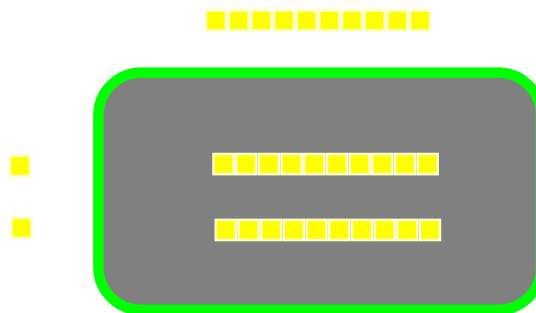
First ask children to make  $2 \times 9$ . They will make this rectangle.

Then ask them to make  $2 \times 10$ - two times ten, or ten taken twice. They may add one cube in each line to make this rectangle.

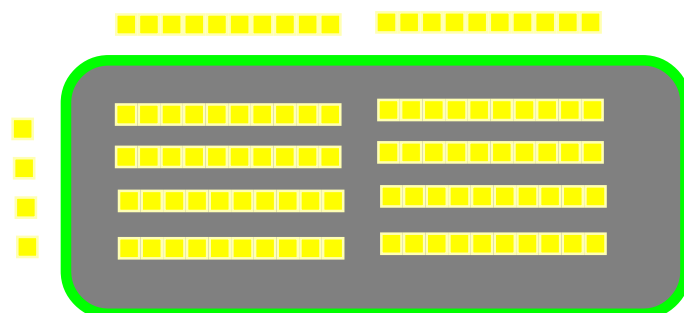


Children know that when they get ten cubes they can make a rod by joining them. Therefore in the above arrangement of  $2 \times 10$ , children join the cubes horizontally to make a rod. If they join the reference cubes to make a rod, they must also do the same to the 'answer' cubes.

$2 \times (1 \text{ ten})$   
Answer: Two tens



$4 \times (2 \text{ tens})$   
Answer : 8 tens

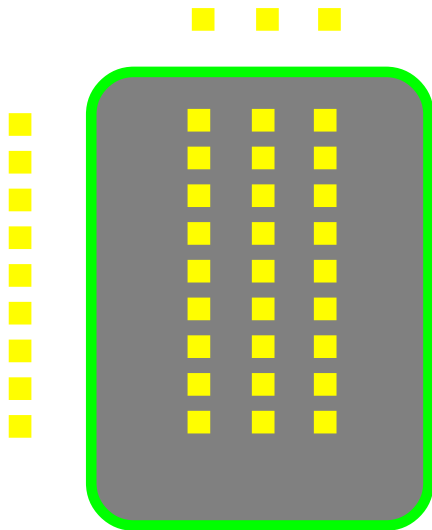


This way children discover the rule of  
 $\text{UNITS} \times \text{TENS} = \text{TENS}$

## TENS TIMES UNITS IS TENS

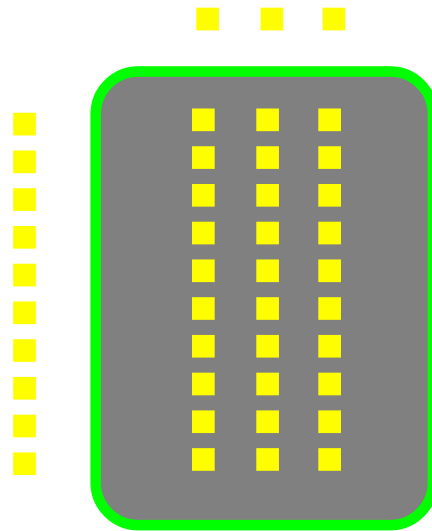
First ask  $9 \times 3$ .

Children will make this rectangle.

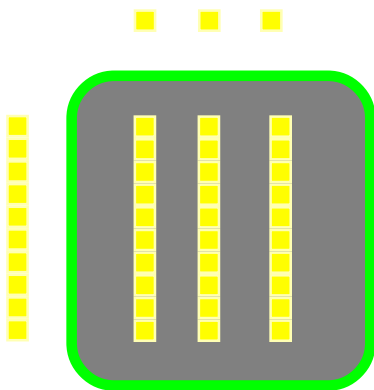


Then ask  $10 \times 3$

They will add one cube to each column and will make this one.



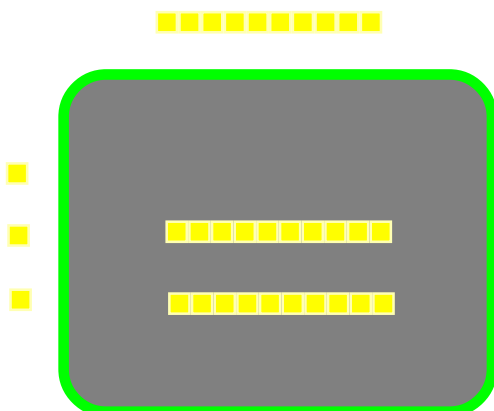
Children know that when they get ten cubes they can make a rod by joining them. Therefore in the above arrangement of  $10 \times 3$ , children join the cubes vertically to make rods.



This is how children discover the rule

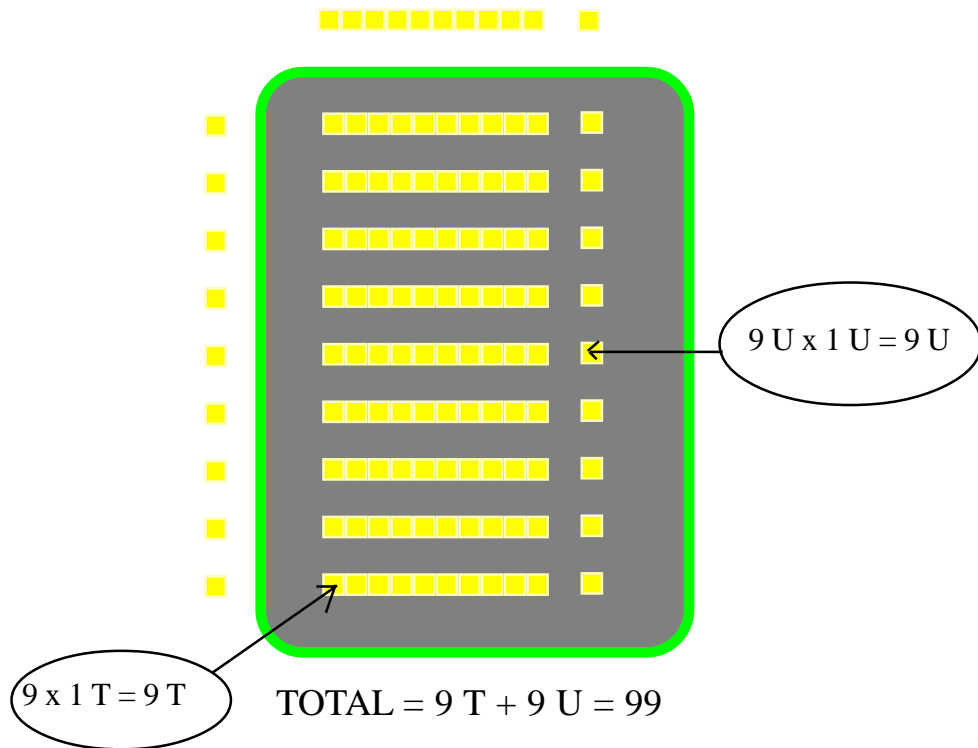
$$\text{TENS} \times \text{UNITS} = \text{TENS}$$

Ask them to compare ( 1 ten x 3 units) with (3 units x 1 ten) . Let them discuss among themselves the similarities and differences.

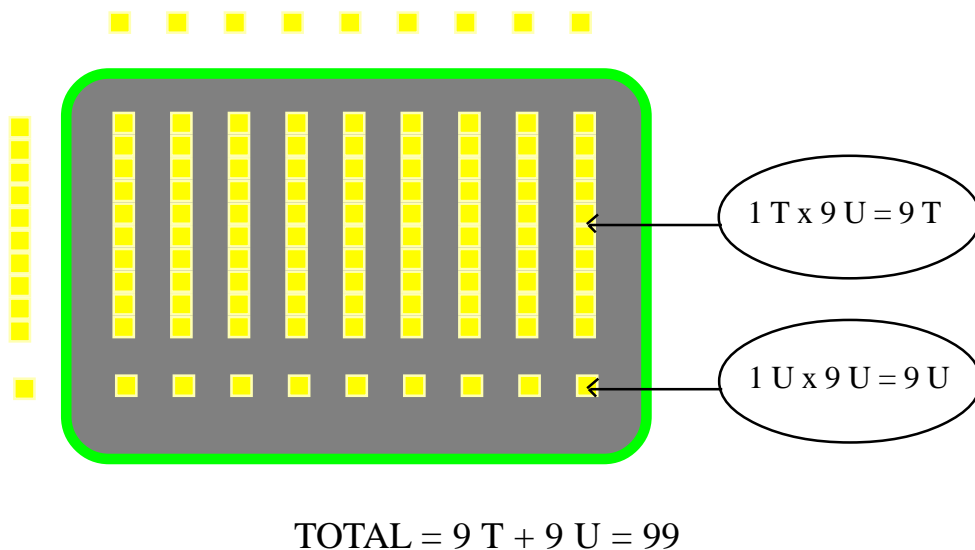


## USING THE DISCOVERED RULES TOGETHER

- \* What is  $9 \times 11$ ?  
 It is 9 times  $(10 + 1)$   
 Which is 9 times a ten and nine times a unit.



- \* What is  $11 \times 9$ ?  
 It is nine units taken ten times, and then once more.  
 It is a ten times nine and a units times nine.

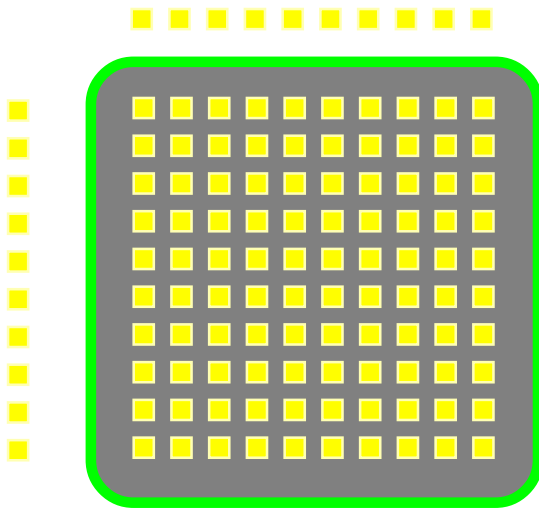


## TENS TIMES TENS ARE HUNDREDS

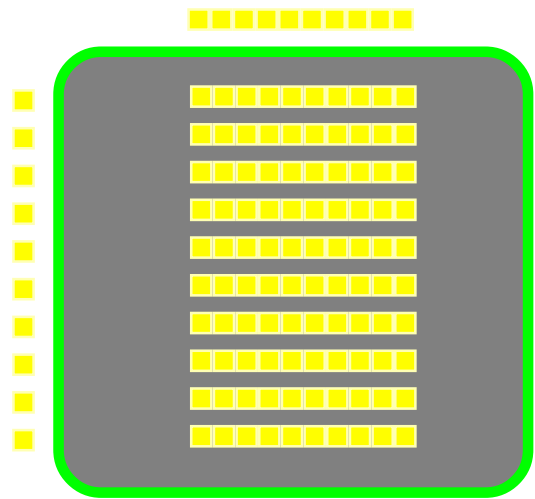
Ask children 'what is  $10 \times 10$ '?

Children will do it in different ways. Each of the different ways can then be joined to make a hundred plate.

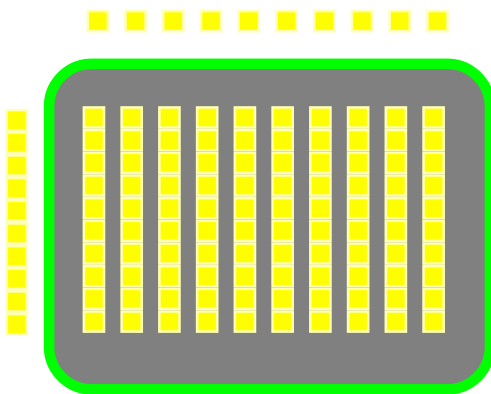
10 units x 10 units



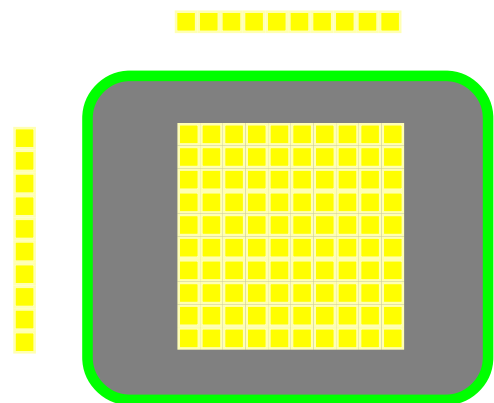
10 units x 1 ten



1 ten x 10 units



1 ten x 1 ten



By making squares of  $10 \times 10$  in different ways, comparing each other's ways and discussing children discover the rule :

$$T \times T = H$$

Now children have discovered four important rules :

$$U \times U = U$$

$$U \times T = T$$

$$T \times U = T$$

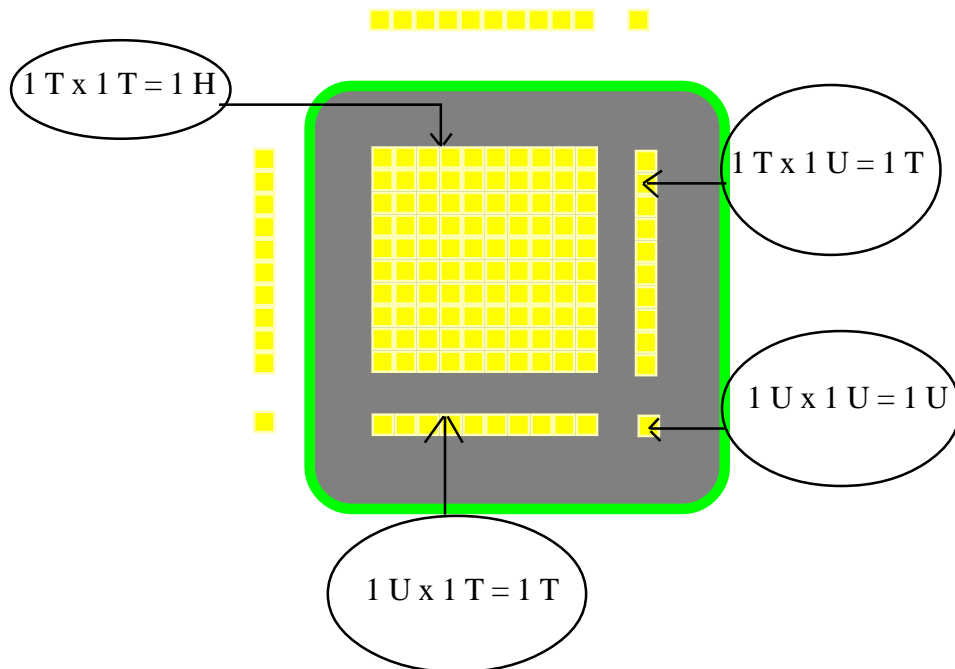
$$T \times T = H$$



## Two Digit x Two digit

This understanding is now applied to solve a problem like 11 x 11 ( two digit x two digit)

11 x 11 :

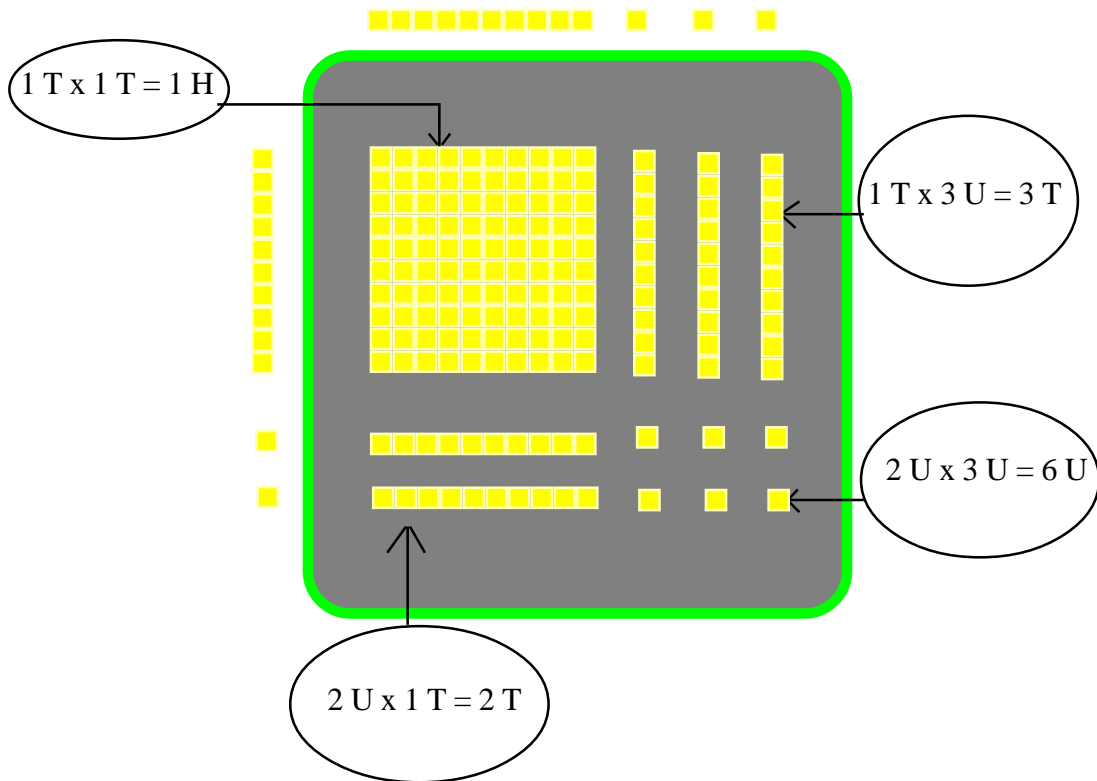


After 10 x 10, only ask what is 11 times 11. Children will struggle, will try out and will discover this square. Here the child has discovered that

$$\begin{aligned} 11 \times 11 &= (10 + 1) \times (10 + 1) \\ &= (10 \times 10) + (10 \times 1) + (1 \times 10) + (1 \times 1) \end{aligned}$$

We should not teach or mention this as a rule at this age. We should only give them the problem and when they make this square they have discovered it.

\* 12 x 13

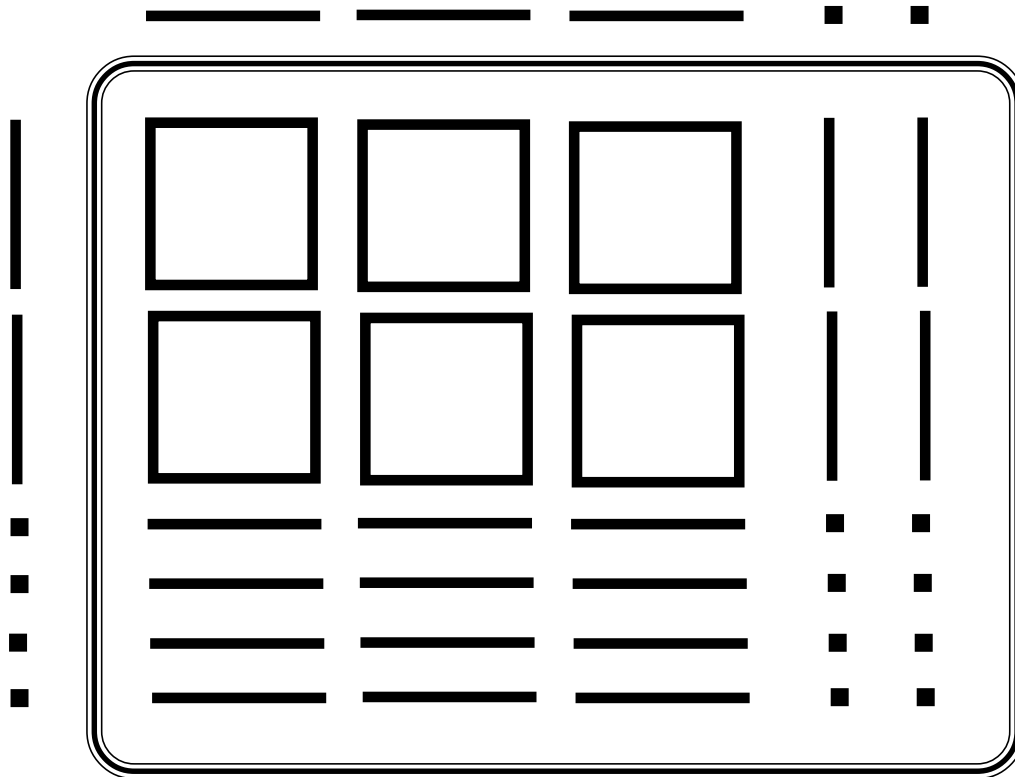


When children become comfortable in making rectangles and getting answers they have understood multiplication in 'Things Language'. This understanding is then translated into pictorial language where they represent multiplication using pictures. Then it is systematically translated into an alphanumeric metric representation and then finally into the standard representation.

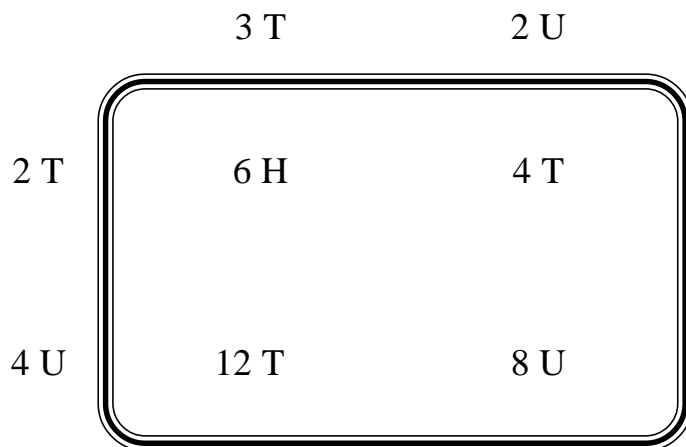
Systematic translation into Pictorial Language :

\* 24 x 32

Children first make rectangle using plates, rods and cubes (UTH kit or Dienes' blocks) . Referring to the arrangement they draw picture as follows :



\* The above picture is then represented as alphanumeric matrix :



\* From the matrix they translate it to the following grid in order to count the total number of hundreds, tens and units.

	H	T	U
	6	4	
+	1	2	8
	7	6	8

SWITCHING OVER TO STANDARD WAY OF WRITING MULTIPLICATION

Children have discovered the rules. That understanding is used to arrive at this.

	H	T	U
		3	2
×		2	4
			8
	1	2	
		4	
+	6		
	7	6	8

2 units taken 4 times gives 8 units.

3 tens taken 4 times gives 12 tens which is equal to 1 hundred and 2 units.

2 units taken 2 T times is 4 tens.

3 tens taken 2 T times gives 6 hundreds.

All added together give 768.