Signed Integers and Their Operations

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Navnirmiti
Universal Active Math

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Introduction :

+ and - signs have two meanings.

Meaning 1 :

+ and - when attached to a number is a property of the number.

+5 means : A person has 5 rupees.

-5 means : A person owes 5 rupees.

+ means having that amount

- means having a loan of that amount.

Thus the + or - sign is the property of that amount, *dhan or Runa*, asset or loan.

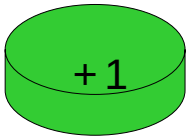
Meaning 2 :

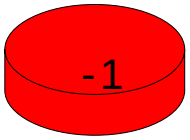
+ and - as defining operations of addition or subtraction to or from an account.

+ means adding to an account.

- means taking away from an account.

Account : A tray on the table defines the account.





A Green plug is for a positive amount of one rupee and a red for a loan of one rupee.

Substitute for Green and red plugs :

You can use carom coins, white and black.

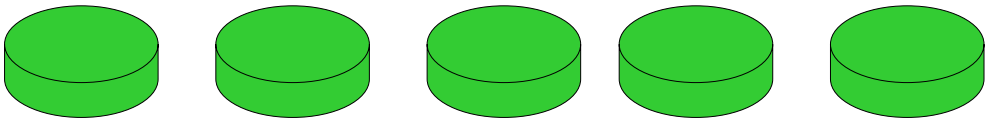
At a place like Selu in Parbhani district of Maharashtra, neither of the above options were available.

We used one-rupee coins to represent positive numbers and broken coconut shell pieces to represent negative numbers.

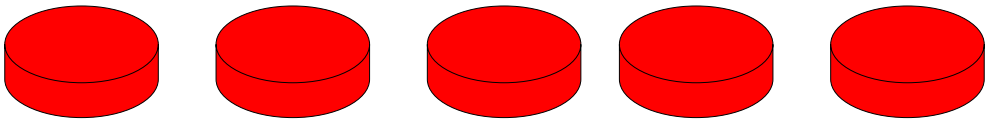
in Marathi this is called 'karvanti' and is used to represent 'begging'.

Quite appropriate!

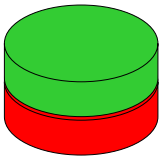
I have +5 rupees. Take 5 green plugs in the tray.



I have -5 rupees. I owe 5 rupees. I have a loan of 5 rupees. Take 5 red plugs.



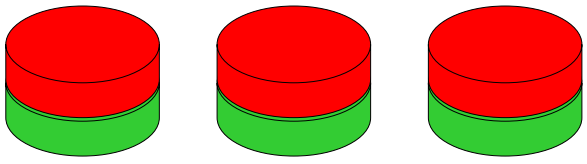
What if I have one rupee and a loan of one rupee?
A plus 1 and a minus 1?



The one rupee will be used to repay the loan and I will be left with zero. (We can say that +1 and -1 cancel each other). Thus I have zero.

A green plug and a red plug kept on each other is a zero.

Here I have 3 zeros, which is equal to having nothing.



Having zero can be represented as 'nothing in the tray' or (+1 and -1) in the tray, or (+50 and -50) in the tray .... any equal number of + and -.

I have +5 rupees and +3 rupees.

This is equal to having +8 rupees.

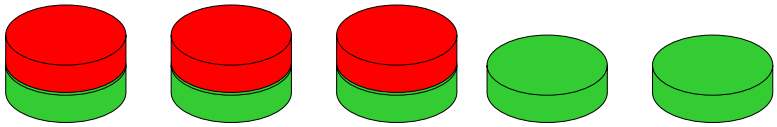
+5 +3 = +8

I have -5 rupees and -3 rupees.

This is equal to having -8 rupees.

-5 -3 = -8

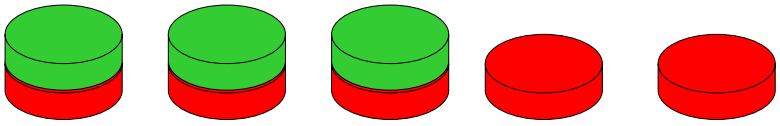
I have +5 rupees and -3 rupees.



5 green and 3 red. 3 pairs making zeros.
This is equal to having +2 rupees.

+5 -3 = +2 or -3 +5 = +2

I have -5 rupees and +3 rupees.



5 red and 3 green. 3 pairs making zeros.
Is equal to having -2 rupees.

-5 +3 = -2 or +3 -5 = -2

Comparisons :

Give some (only green) / (only red) / (green & red) plugs to each child.

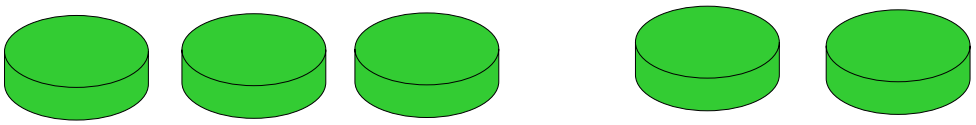
Ask the following questions :
- How much do you have in your account?
- Who has more money ?
- Who has less money?
- Who has more loan?
- Who has less loan? etc.

Show two accounts in two hands :

One having 4 green and 2 red plugs and the other having 3 green plugs. Ask the child which one he/she would like to have.

Addition :

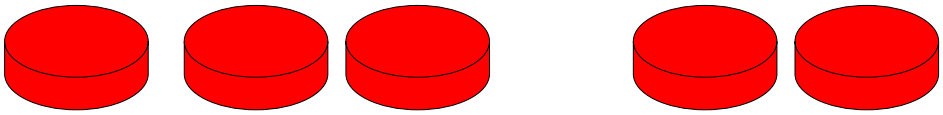
(+3) + (+2) Take 3 rupees in the tray
Add
2 rupees



Total 5 rupees.

(+3) + (+2) = (+5)

(-3) + (-2) Take a loan of 3 rupees in the tray
Add
a loan of 2 rupees



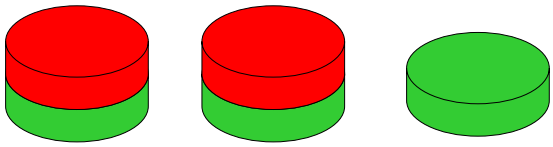
Total loan of 5 rupees.

(-3) + (-2) = (+5)

(+3) + (-2) Take 3 rupees in the tray

Add

a loan of 2 rupees



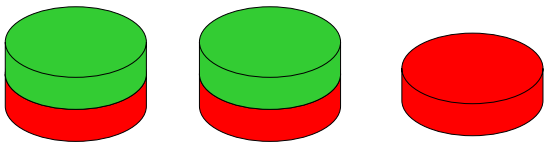
A loan of 2 rupees makes pairs with an amount of 2 rupees giving 2 zeros. Effectively there is 1 rupee.

(+3) + (-2) = (+1)

(-3) + (+2) Take a Loan of 3 rupees in the tray

Add

2 rupees



An amount of 2 rupees makes pairs with a loan of 2 rupees giving 2 zeros. Effectively there is loan of 1 rupee.

(-3) + (+2) = (-1)

(-5) + (+2) = (-3)

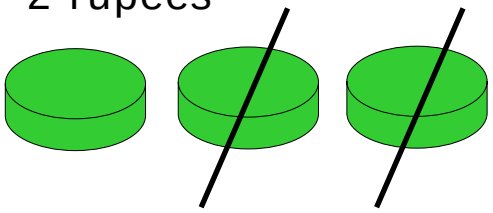
Subtraction :

Minus means take away.

(+3) - (+2) Take 3 rupees in the tray

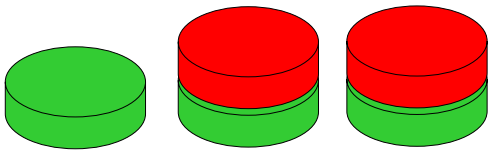
Take away

2 rupees



1 rupee left in the tray.

This is also equivalent to : +3 -2

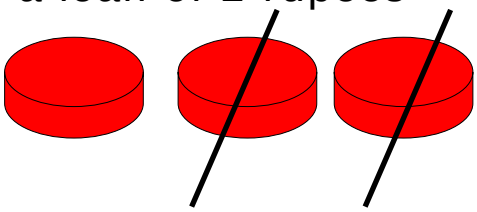


(+3) - (+2) = +3 -2 = +1

(-3) - (-2) Take a Loan of 3 rupees in the tray

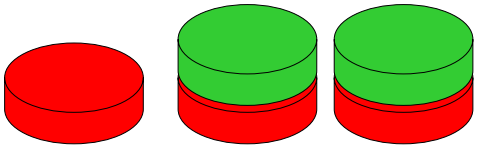
Take away

a loan of 2 rupees



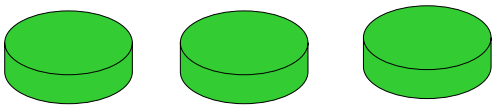
A loan of 1 rupee left in the tray.

This is also equivalent to : -3 +2

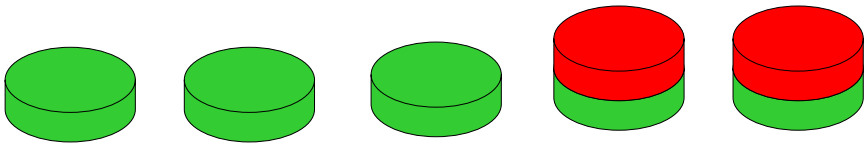


(-3) - (-2) = -3 +2 = -1

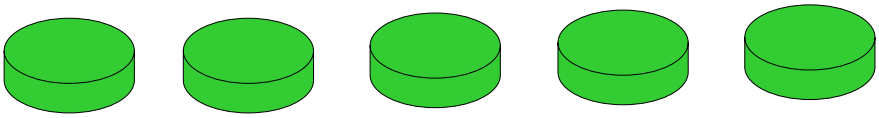
(+3) - (-2) Take 3 rupees in the tray



Take away a loan of 2 rupees.
But there are no 2 red plugs in the tray.
To get them we add two zeros in the tray.
Now we have the following in the tray.



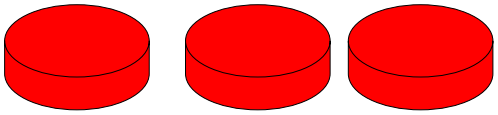
Take away a loan of 2 rupees.



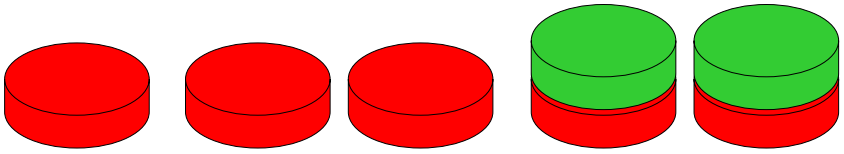
Now the tray has 5 green plugs.
The answer is (+5).

(+3) - (-2) = (+5)

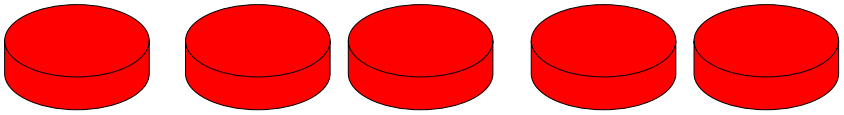
(-3) - (+2) Take a Loan of 3 rupees in the tray.



We have to take away 2 rupees.
But there are no 2 green plugs in the tray.
To get them we add two zeros in the
tray. Now we have the following in the tray.



Take away 2 rupees.



Now the tray has 5 red plugs, a loan of
5 rupees. The answer is (-5).

(-3) - (+2) = (-5)

Multiplication :

Nothing in the tray to begin with. We take a given number a certain number of times, and arrive at something in the tray.

(+2) x (+3) Two times (+3)

3 rupees to be taken in the tray 2 times

6 rupees

(+2) x (+3) = (+6)

(+2) x (-3) (Plus two) times (minus three).

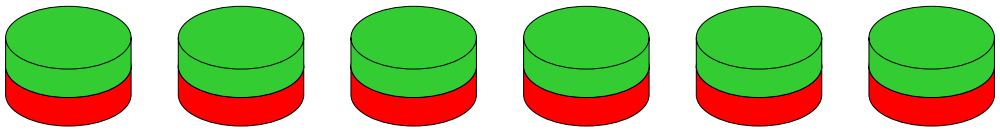
A loan of 3 rupees is to be taken in the tray 2 times.

(-6) rupees.

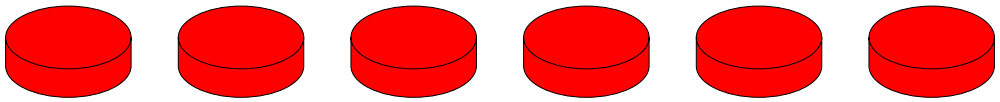
(+2) x (-3) = (-6)

Multiplying by a negative number means taking away those many times.
Nothing in the tray to begin with.

(-2) x (+3) (minus two) times (plus three).
3 rupees are to be taken away 2 times.
But there is nothing in the tray.
So we take 6 zeros.



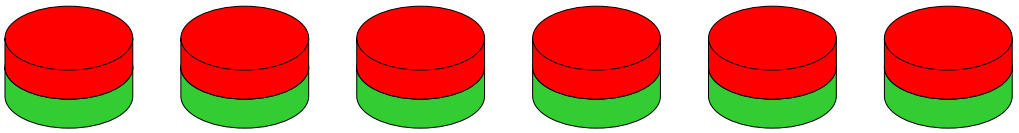
Now take away 3 rupees 2 times.



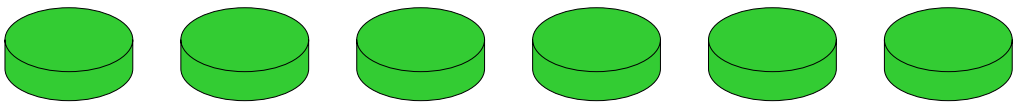
The tray has -6.

(-2) x (+3) = (-6)

(-2) x (-3) (minus two) times (minus three).
A loan of 3 rupees is to be taken away 2 times. But there is nothing in the tray.
So we take 6 zeros in the tray.



Now take away a loan of 3 rupees 2 times.



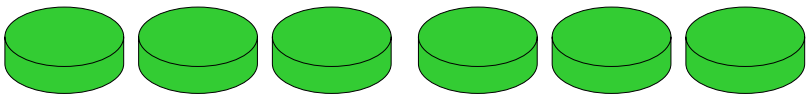
The tray has 6.

(-2) x (-3) = (+6)

Division :

When we have to divide a number, we start with that number in the tray. Dividing by a number means making those many equal parts of the number in the tray. If we remove those parts one by one we end up with nothing in the tray. Division is therefore the opposite of multiplication. Multiplication is repeated addition of the same number. Division is repeated subtraction of a given number.

(+6)/2 Start with (+6) in the tray.



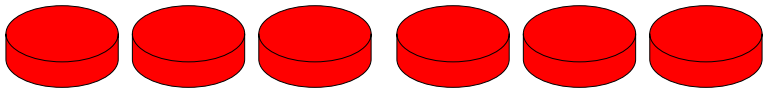
By the meaning of equal sharing, divide them equally among two children.

Each child gets (+3). The answer is (+3).

Nothing left in the tray.

(+6) / (+2) = (+3)

(-6) / 2 Start with (-6) in the tray.



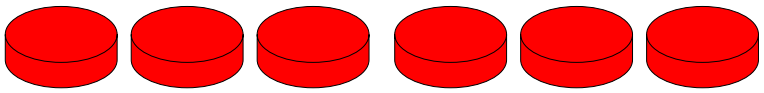
By the meaning of equal sharing, divide them equally among two children.
Each child gets (-3). The answer is (-3).

Nothing left in the tray.

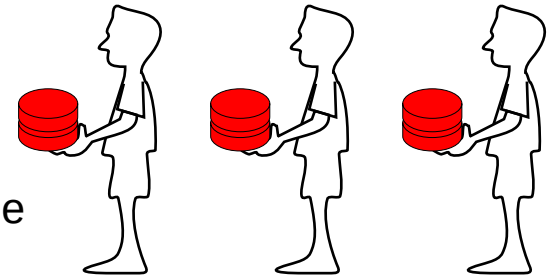
(-6) / (+2) = (-3)

(-6) / (-2)

Start with -6 in the tray.



By the meaning of equal grouping,
How many (-2)s are there in (-6)?
If you give (-2) to each child, how many children will get it? The answer is 3.



Nothing left in the tray.

(-6) / (-2) = (+3)

Dividing by a negative number :

Dividing by (+2) is taking away equal portions twice so that we are left with nothing in the tray.

Dividing by (-2) is putting equal portions twice in the tray so that we are left with nothing in the tray.

This can happen in two ways.

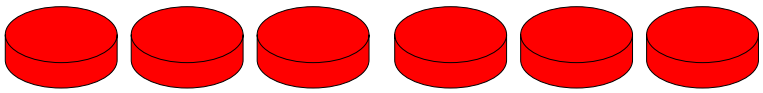
If I have a loan in the tray, I can put in a positive amounts and be left with nothing in the tray.

If I have positive amount in the tray, I can put in loan and be left with nothing in the tray.

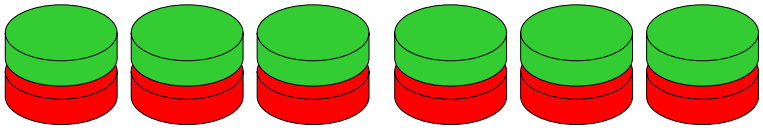
Example :

(-6)/(-2)

Start with -6 in the tray.



Put equal portions twice so that I will be left with nothing in the tray. I put (+3) twice.



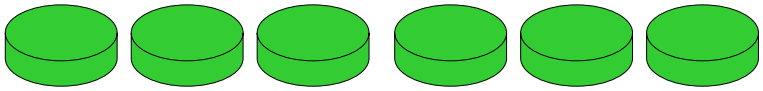
The quantity that I put twice is the answer.

(-6)/(-2) = (+3)

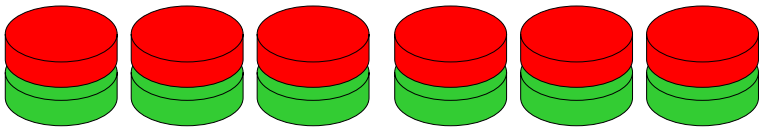
Example :

(+6)/(-2)

Start with +6 in the tray.



Put equal portions twice so that I will be left with nothing in the tray. I put (-3) twice.



The quantity that I put twice is the answer.

(+6)/(-2) = (-3)

I Do, I Understand !